# Swansea-Tianjin Stochastic Analysis Workshop

June 27-28, 2024

Swansea University

| 27 June | Title | |
|---|---|---|
| 9:00-9:15 | Welcome Speech Vitaly Moroz (Head of Department) | |
| Chair | Eugene Lytvynov | |
| 9:15-10:00 | Feng-Yu Wang | Wasserstein Convergence Rate for Empirical Measures of Markov Processes |
| 10:00-10:45 | Angelica Pachon | Upper bounds for the largest components in critical inhomogeneous random graphs |
| 10:45-11:15 | Tea Break | |
| Chair | Jinghai Shao | |
| 11:15-12:00 | Dmitri Finkelshtein | Spatio-Temporal Correlations in a Class of Interacting Particle Systems |
| 12:00-12:45 | Eryan Hu | The existence and on-diagonal upper bound of the heat kernel on doubling spaces |
| 12:45-14:00 | Lunch | |
| Chair | Dmitri Finkelshtein | |
| 14:00-14:45 | Xing Huang | Quantitative Propagation of Chaos for Mean Field Interacting Particle System |
| 14:45-15:30 | Goncalo Dos Reis | High order splitting methods for stochastic differential equations |
| 15:30-15:45 | Tea Break | |
| Chair | Angelica Pachon | |
| 15:45-16:30 | Huaiqian Li | Wasserstein Convergence Rates for Empirical Measures of Subordinated Fractional Brownian Motions |
| 16:30-17:15 | Jiawei Li | Mean field equations arising from random vortex dynamics |
| 17:15-18:00 | Zeev Sobol | Revised Einstein model: Chemotaxis and Self-control |

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| 18:00 | Dinner |

| 28 June | Title | |
|---|---|---|
| Chair | Chenggui Yuan | |
| 9:00-09:45 | William Salkeld | Locally interacting equations and their associated McKean-Vlasov marginal |
| 09:45-10:30 | Michael Hinz | Variability of mappings and differential equations with BV-coefficients |
| 10:30-11:15 | Niels Jacob | Decomposing generators of nonlocal symmetric Dirichlet forms and monotone operators in the sense of Browder and Minty |
| 11:15-11:30 | Tea Break | |
| Chair | Feng-Yu Wang | |
| 11:30-12:15 | Jinghai Shao | Averaging principle for two time-scale regime-switching processes |
| 12:15-13:00 | Eugene Lytvynov | Determinantal point processes and quasi-free states on the CAR algebra |
| 13:00-14:30 | Lunch | |
| 14:30-16:00 | Discussion | |

# Abstract

**Wasserstein Convergence Rate for Empirical Measures of Markov Processes**

Feng-Yu Wang
Swansea University

**Abstract**
We prove a general result on the convergence rate in Wasserstein distance for empirical measures of Markov processes. In particular, sharp uniform Wasserstein convergence rate is presented for the empirical measures of ergodic Markov processes with common invariant probability measure satisfying certain conditions. As applications, the main result is illustrated by subordinations of some typical models excluded by existing results: stochastic Hamiltonian systems, spherical velocity Langevin processes, multi-dimensional Wright-Fisher type diffusion processes, and stable type jump processes.

**Upper bounds for the largest components in critical inhomogeneous random graphs**

Angelica Pachon
Swansea University

**Abstract**
We consider the Norros-Reittu random graph NRn(w), where edges are present independently but edge probabilities are moderated by vertex weights, and use probabilistic arguments based on martingales to analyse the component sizes in this model when considered at criticality. In particular, we obtain stronger upper bounds (with respect to those available in the literature) for the probability of observing unusually large maximal clusters, and simplify the arguments needed to derive polynomial upper bounds for the probability of observing unusually small largest components.
This is a joint work with Umberto De Ambroggio

**Spatio-Temporal Correlations in a Class of Interacting Particle Systems**

Dmitri Finkelshtein
Swansea University

**Abstract**
For interacting particle systems in the continuum, the study of spatial pair correlations between particles at different spatial positions is well-established, typically using second-order (spatial) correlation functions which represent factorial spatial moments of the system's states. However, the investigation of spatio-temporal pair correlations between particles located at different spatial positions in different moments in time has received relatively little attention and lacks a comprehensive approach. We

introduce a general method suitable for a broad class of non-diffusive interacting particle systems in the continuum. Our approach simplifies the analysis of spatio-temporal correlations by relating them to the study of spatial correlations in auxiliary multi-type systems. We demonstrate our approach for several population dynamics and consider their mean-field and beyond-mean-field behaviour. Finally, we validate our theoretical predictions by comparing them to the results obtained through computer simulations. Based on a joint paper with Otso Ovaskainen and Panu Somervuo.

**The existence and on-diagonal upper bound of the heat kernel on doubling spaces**

Eryan Hu
Tianjin University

**Abstract**

In this talk, we will first recall some methods on how to obtain the existence of heat kernels. Then, we introduce a new approach to investigate the on-diagonal upper bound of heat kernels for regular non-local Dirichlet forms on metric measure spaces with volume doubling condition. As hypotheses, we use the Faber-Krahn inequality, the generalized capacity condition and an upper bound for the integrated tail of the jump kernel. The parabolic mean value inequality for subcaloric functions plays an important role. This talk is based on joint works with Professor Alexander Grigor'yan (Bielefeld University, Germany) and Professor Hu (Tsinghua University, China).

**Quantitative Propagation of Chaos for Mean Field Interacting Particle System**

Xing Huang
Tianjin University

**Abstract**

In this talk, quantitative propagation of chaos in $L^\eta$($\eta\in(0,1)$)-Wasserstein distance for mean field interacting particle system is derived, where the diffusion coefficient is allowed to be interacting and the initial distribution of interacting particle system converges to that of the limit equation in $L^1$-Wasserstein distance. The non-degenerate and degenerate cases are investigated respectively and the main tool relies on the gradient estimate of the decoupled SDEs.

**High order splitting methods for stochastic differential equations**

Goncalo Dos Reis
University of Edinburgh

**Abstract**

In this talk, we will discuss how ideas from rough path theory can be leveraged to develop high order numerical methods for SDEs. To motivate our approach, we consider what happens when the Brownian motion driving an SDE is replaced by a piecewise linear path. We show that this procedure transforms the SDE into a sequence of ODEs – which can then be discretized using an appropriate ODE solver. Moreover, to achieve a high accuracy, we construct these piecewise linear paths to

match certain “iterated” integrals of the Brownian motion. At the same time, the ODE sequences obtained from this path-based approach can be interpreted as a splitting method, which neatly connects our work to the existing literature. For example, we show that the well-known Strang splitting falls under this framework and can be modified to give an improved convergence rate. We will conclude the talk with a couple of examples, demonstrating the flexibility and convergence properties of our methodology.
(joint work with James Foster and Calum Strange, https://arxiv.org/abs/2210.17543, https://epubs.siam.org/doi/10.1137/23M155147X)

### Wasserstein Convergence Rates for Empirical Measures of Subordinated Fractional Brownian Motions

Huaiqian Li
Tianjin University

**Abstract**
In this talk, I will consider empirical measures associated with subordinated fractional Brownian motions on the flat torus. I will talk about rates of convergence for Wasserstein distances $W_p$ between empirical measures and the uniform measure on the flat torus. As an application of the main result, Wasserstein convergence rates for discrete time subordinated fractional Brownian motions are provided. Main ideas will be explained.

### Mean field equations arising from random vortex dynamics

Jiawei Li
University of Edinburgh

**Abstract**
We consider Mckean-Vlasov type stochastic differential equations with multiplicative noise arising from the random vortex method. Such an equation can be viewed as the mean-field limit of interacting particle systems with singular interacting kernels such as the Biot-Savart kernel. A new estimate for the transition probability density of diffusion processes will be formulated to handle the singularity of the interacting kernel. The existence and uniqueness of the weak solution of such SDEs will be established as the main result. Based on the joint work with Zhongmin Qian (Oxford).

### Revised Einstein model: Chemotaxis and Self-control

Zeev Sobol
Swansea University

**Abstract**
This is a further development of a revised Einstein model of “the movement of small particles in a stationary liquid”, demonstrating a finite propagation speed. This time, it involves chemotaxis with a non-dissipating reagent, and a self-control element of a drive to an optimal concentration. We study conditions such that 1dim dynamics of the concentration, demonstrates travelling waves.

## Locally interacting equations and their associated McKean-Vlasov marginal

William Salkeld
University of Nottingham

**Abstract**

In this talk, I will discuss the dynamics of a collection of Gaussian stochastic differential equations indexed by a locally finite graph. The drift of each individual equation is dependent only on the dynamics of the individual and their neighbours so that each SDE exhibits strong correlation with a small number of other SDEs via these local interactions.

Such local interactions arise in statistical physics, engineering and simulation of SPDEs, and are suitable for applications when long range interactions between distant individuals are described via a sequence of local interactions between neighbours. We will focus on simulation of the Allen-Cahn equation.

The price we pay for considering local interactions instead of macroscopic 'mean-field' interactions is that the when the number of equations is large we do not expect the statistical decoupling of any pair of equations. Instead, the dynamics exhibit a 'Markov Random Field' property where equations are conditionally independent of one another conditioning on an appropriate separating subset.

Further, these systems of equations are continuously dependent on the underlying graph structure so that changes in the interactions leads to proportional changes in the all equations.

These fundamental properties of the entire system of equations is key to understanding the microscopic dynamics of each individual equation, and deriving a McKean-Vlasov equation that describes the marginal distribution of local neighbourhoods.

## Variability of mappings and differential equations with BV-coefficients

Michael Hinz
Bielefeld University

**Abstract**

Given two finite Borel measures on $\mathbb{R}^n$, the finiteness of their mutual Riesz (interaction) energy of a certain order means that they cannot be too concentrated at the same spots. If one of these measures is the occupation measure of a H\"older continuous or low Sobolev regular mapping $u$ and the other is the gradient measure of a BV-function $\varphi$, then the finiteness of the mutual Riesz energy implies that the nonlinear composition $\varphi\circ u$ is well defined and of a certain Sobolev regularity. We briefly

discuss some first applications of this principle to pathwise integrals in the Young regime and related stochastic differential equations with fractional Brownian driving. If time permits, we briefly mention some related variational problems. The talk is based on joint work with Jonas

T\"olle and Lauri Viitasaari (both Aalto University).

**Decomposing generators of nonlocal symmetric Dirichlet forms and monotone operators in the sense of Browder and Minty**

Niels Jacob
Swansea University

**Abstract**

The classical Dirichlet form can be written as the integral over the the square of the square root of the Laplacian applied to suitable functions, but it can also be represented as the integral over the square of the gradient applied to such functions. In the first case the corresponding p-energy is easily identified with operators related to non-linear potentials in the sense of Maz'ya and Havin, in the second case we encounter the p-Laplacian, the Riesz transform shows the connection of these two approaches. We will consider certain classes of pseudo-differential operators with negative definite functions as symbols and will discuss decomposition which are in some sense analogous to the one mentioned for the Laplacian. But in certain aspects the situation is much more involved and leads to some challenging, so far not yet solved problems, e.g. Fourier multipliers for non smooth symbols. The key tool is to identify the p-energy as Gateaux derivative of the p-energy evaluated at u applied to u. This Gateaux derivative is a monotone operator in the sense of Browder and Minty, if we consider, as we do in case of capacities, the p-energy plus the p-norm to the power p, this operator is even strictly monotone, hence has strictly monotone inverse which shall lead to a potential operator. Studying these different decompositions of nonlocal anisotropic Dirchlet operators shall lead to a better insight on the effects of the anisotropic behaviour.

**Averaging principle for two time-scale regime-switching processes**

Jinghai Shao
Tianjin University

**Abstract**

This work studies the averaging principle for a fully coupled two time-scale system, whose slow process is a diffusion process and fast process is a purely jumping process on an infinitely countable state space. The ergodicity of the fast process has important impact on the limit system and the averaging principle. We show that under strongly ergodic condition, the limit system admits a unique solution, and the slow process converges in the L_1-norm to the limit system. However, under certain weaker ergodicity condition, the limit system admits a solution, but not necessarily unique, and the slow process can be proved to converge weakly to a solution of the limit system.

**Determinantal point processes and quasi-free states on the CAR algebra**

Eugene Lytvynov
Swansea University

**Abstract**

A quasi-free state over the algebra of the canonical anticommutations relations (CAR)

is a state with respect to which the moments of the field (Segal-type) operators are calculated similarly to the expectation of a Gaussian random field (when one additionally takes into account the sign of a partition). An important subclass of quasi-free states is given by gauge-invariant states. For a given representation of the CAR, one formally defines its particle density as the product of the creation and annihilation operators at point, and again formally the smeared particle density is a family of commuting Hermitian operators. For a class of quasi-free states, we show that its particle density can be rigorously realised as a family of commuting self-adjoint operators and its joint spectral measure is a determinantal point process, i.e., a point process whose correlation functions are determinants built upon a correlation kernel $K(x,y)$. In the case of a gauge-invariant quasi-free state, the correlation kernel $K(x,y)$ is Hermitian. We also consider the particle-hole transformation in the continuum as a certain Bogoliubov transformation of a gauge-invariant quasi-free state, which leads to a non-gauge-invariant quasi-free state. For the corresponding particle density, the joint spectral measure is a determinantal point process with a correlation kernel $K(x,y)$ that is J-Hermitian. The latter means that the integral operator with integral kernel $K(x,y)$ is self-adjoint with respect to an indefinite inner product.