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Quantum Riemannian geometry of the finite chain A_n graph, jets and geodesics

Abstract

We outline the bimodule formalism of noncommutative Riemannian geometry, where the coordinate algebra can be any unital algebra A equipped with a differential structure expressed as an A-bimodule Ω^1 of 1-forms. As an application, we show how to write down quantum gravity path integral models on finite-dimensional algebras or algebras which are graded with finite-dimensional components such as the fuzzy sphere. When A is functions on a discrete set, the choice of Ω^1 is the same thing as a directed graph, where arrows are 1-forms. As an example, we find that the intrinsic quantum Riemannian geometry of the A_n graph $\circ-\circ-\ldots-\circ$ of n vertices is necessarily q-deformed with $q^{2(n+1)}=1$, even though there is no quantum group in sight. We also see how solutions of the Yang–Baxter or braid relations arise naturally from noncommutative differential geometry and relate both to quantum jet bundles and to a notion of quantum geodesic flows.