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The Dirichlet–Ferguson diffusion on the space of probability measures over a closed Riemannian manifold

Abstract

We construct a diffusion process on the L^2 -Wasserstein space $P_2(M)$ over a closed Riemannian manifold M . The process, which may be regarded as a candidate for the Brownian motion on $P_2(M)$, is associated with the Dirichlet form induced by the L^2 -Wasserstein gradient and by the Dirichlet–Ferguson random measure with intensity the Riemannian volume measure on M . We discuss the closability of the form via an integration-by-parts formula, which allows explicit computations for the generator and a specification of the process via a measure-valued martingale problem. We comment how the construction is related to previous work of von Renesse–Sturm on the Wasserstein Diffusion, of Kondratiev–Lytvynov–Vershik on diffusions on the cone of Radon measures, and of Konarovskiy–von Renesse on the Modified Massive Arratia Flow. (Ann. Probab. 50(2):591–648, 2022)